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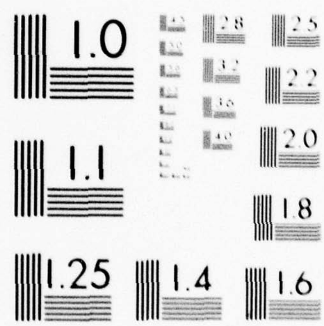

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Isaacs B. S.

6 Finite Fourier Filters,

by

10 James C. Isaacs, Jr.

Advanced Computer Laboratory
Computer Department

12 37 p'

Melpar Incorporated
3000 Arlington Boulevard
Falls Church, Virginia
(P401-288-10)

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Introduction

Conventional Fourier series analysis enables one to extract information from a periodic function of time. In practical situations, however, one often deals with finite time functions, where conventional Fourier series techniques are inadequate. Finite functions result from a singular or finite set of disturbances. The transient response of a circuit is an example of such a disturbance.

Often one deals with network synthesis problems where it is desirable to describe the output and input functions of the unknown networks. Laplace transform techniques often fail when the output of the network is a finite function; this is because the Laplace transform of the finite function results in a transcendental function in the complex frequency variable. Further, one cannot express the finite output as a conventional Fourier series to obtain a rational Laplace transform of the output.

The theory incorporated in finite Fourier filters enables one to deal with the type of time functions mentioned above. This technical note attempts to summarize, in a logical development, the basic theory of finite Fourier analysis, and the synthesis which incorporates the theory in the finite Fourier filters.

Finite Fourier Filters

I. Fourier Analysis of a Periodic Wave

Given a time dependent periodic function, $F(t)$, it is possible to represent this function exactly with the familiar Fourier series

$$F(t) = A_o + \sum_{n=1}^{\infty} \left(A_n \cos \frac{n\pi}{T} t + B_n \sin \frac{n\pi}{T} t \right) \qquad (1)$$

The sine and cosine functions of this series comprise an orthogonal set, i.e.,

(2)

(a) $\int_0^{2T} [f_1(t)]_n \, [f_2(t)]_{n'} \, dt = 0$ for all integer values of n and n'

(b) $\int_0^{2T} [f_1(t)]_n \, [f_1(t)]_m = k$ (a constant) if $n = m$; $= 0$ if $n \neq m$ — m is some multiple of n

(c) $\int_0^{2T} [f_2(t)]_{n'} \, [f_2(t)]_{m'} = k'$ (a constant) if $n' = m'$; $= 0$ if $n' \neq m'$ — m' is some multiple of n'

where:

$[f_1(t)]_n$ corresponds to cosine terms,

$[f_2(t)]_{n'}$ corresponds to sine terms, and

$2T$ = period of $[f_1(t)]_n$ and $[f_2(t)]_{n'}$ for $n = n' = 1$.

Using the orthogonal properties of equations (2) it is a simple matter to determine the coefficients A_n and B_n for a particular function $F(t)$. For example, to determine A_1 multiply both sides of equation (1) by

$$[f_1(t)]_1 = \cos \frac{(1)\pi}{T} t$$

and integrate over the period 2T,

$$\int_0^{2T} F(t)\cos(\tfrac{\pi}{T})t\,dt = A_0\int_0^{2T}\cos(\tfrac{\pi}{T})t\,dt + A_1\int_0^{2T}\cos(\tfrac{\pi}{T})t\cos(\tfrac{\pi}{T})t\,dt$$

$$+ A_2\int_0^{2T}\cos(\tfrac{2\pi}{T})t\cos(\tfrac{\pi}{T})t\,dt + \cdots + A_n\int_0^{2T}\cos(\tfrac{n\pi}{T})t\cos(\tfrac{\pi}{T})t\,dt$$

$$+ B_1\int_0^{2T}\sin(\tfrac{\pi}{T})t\cos(\tfrac{\pi}{T})t\,dt + B_2\int_0^{2T}\sin(\tfrac{2\pi}{T})t\cos(\tfrac{\pi}{T})t\,dt + \ldots$$

$$+ B_n\int_0^{2T}\sin(\tfrac{n\pi}{T})t\cos(\tfrac{\pi}{T})t\,dt.$$

or

$$\int_0^{2T} F(t)\cos(\tfrac{\pi}{T})t\,dt = 0 + A_1\int_0^{2T}\cos^2(\tfrac{\pi}{T})t\,dt + 0 + \ldots + 0 + 0 + 0 + \ldots + 0$$

According to equation (2).

$$\therefore A_1 = \frac{\int_0^{2T} F(t)\cos(\tfrac{\pi}{T})t\,dt}{\int_0^{2T}\cos^2(\tfrac{\pi}{T})t\,dt}$$

In general,

$$\text{(a)}\quad K_n' = \frac{\int_0^{2T} F(t)\,h_n(t)\,dt}{\int_0^{2T}[h_n(t)]^2\,dt} \qquad (3)$$

where:

K_n' represents either A_n or B_n

h_n (t) represents either cos $(\frac{n\pi}{T})t$ or sin $(\frac{n\pi}{T})t$.

One can normalize the above equation by requiring

$$\int_0^{2T}[h_n(t)]^2\,dt = 1$$

then,

$$\text{(b)}\quad K_n = \int_0^{2T} F(t)\,h_n(t)\,dt \qquad n = 0, 1, 2, 3, \ldots \qquad (3)$$

where $h_o(t) = 1$ and $K_o = A_o$.

The normalized Fourier series can now be written as

(c) $$F(t) = \sum_{n=0}^{\infty} k_n h_n(t)$$

Since the integrals, equations (2b) and (2c), yield a result different from zero only when the two functions under the integrals are the same, one can say that when the integral of (3b) has a value K_n for a particular $h_n(t)$, the normalized function $F(t)$ must contain a harmonic which is the same as $h_n(t)$; further the magnitude of K_n is equal to the ratio of the magnitude of this harmonic wave contained in $F(t)$ to the magnitude of $h_n(t)$.

II. Finite Fourier Analysis

In practical situations, the time function to be analyzed may not be a periodic function. More often than not, the function is finite with respect to time, i.e., the function exists only for a certain duration in time.

The problem is to devise an electrical network which can analyze finite functions of time in terms of the cosine and sine functions of a Fourier series. Such a network might be in the form of a 3 port network as in Figure 1a. The output C_n would be proportional to the integral

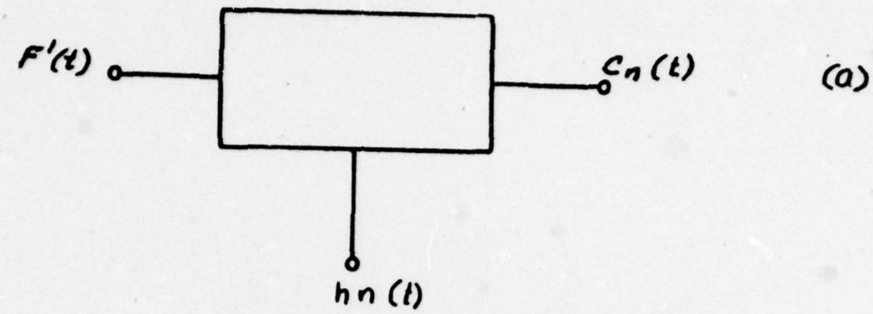
$$\int_0^T F'(t)\, h_n(t)\, dt \qquad (4)$$

where:

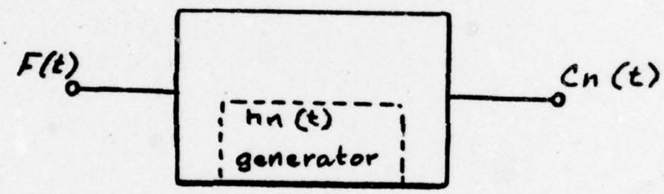
T = half the period of $h_1(t)$

$F'(t)$ = finite function with time duration $\leq T$.

If $F'(t)$ exists for $t > T$, $F'(t)$ is "truncated" into a function $F''(t)$.



3-port Network



2-port Network

Figure 1 Possible networks for analyzing a finite function $F'(t)$ in terms of a Fourier type Series

Multiplying C_n, at time T by the appropriate constant would yield the Fourier coefficient K_n, if $F'(t)$, a finite function, was a periodic odd function of time, $F(t)$ (equation 3b). This can be accomplished by making an odd periodic extension of the finite function $F'(t)$ as shown in Figure 2(a).

One could form the 3-port network into a 2-port network by making $h_n(t)$ an integral generator of the box in Figure 1(b). The 2-port network can be realized as a network synthesis problem if C_n is associated with an output function and $F(t)$ as an input function. Taking the ratio of the Laplace transforms of $F(t)$ (the odd periodic extension of $F'(t)$) and C_n, the ratio of the resulting polynomials in s, $Q(s) = \frac{C_n(s)}{F(s)}$, will allow the formation of a realizable network, provided $Q(s)$ satisfies the Hurtwitz criteria for a positive-real rational function in s[1].

Multiplication in the frequency domain, i.e., $F(s)\, Q(s) = C_n(s)$, corresponds to convolution in the time domain

$$C'_n(x) = G_n(x) * F(x) = \int_0^{\infty} F(t)\, G_n(x-t)\, dt \tag{5}$$

where:

x = a particular value of time $0 \leq x \leq 2T$

$G_n(t)$ is the impulse response of the network whose function in the variable s is $Q(s)$.

$(x-t)$ is a period of time for which $G_n(x-t)$ is integrated with $F(t)$, i.e.,

$G_n(x-t)$ exists for $0 \leq t \leq x$

$G_n(x-t) = 0$ for $t > x$

(1) Guillemin, Synthesis of Passive Networks, 1957, John Wiley & Sons, pgs. 10-36.

The convolution process is illustrated in Figure 2. The function $G_n(t)$, shown as a sine wave, is reflected about the vertical time reference, Figure 2(b), and advanced x seconds. The crossed hatched area, shown in Figure 2(d), is proportional to C_n' at time $t=T-x$. $F(t)$ can be thought of as a weighting function of the area under the curve $G_n(x-t)$ as $G_n(x-t)$ passes from $x=0$ to $x=2T$. However, we are interested in the value C_n' at time $x=T$, Figure 2(d), since the value of the integral, equation 5, is proportional to the desired Fourier coefficient at this instant.

If equation (5) is compared with equation (3b),

$$C_n'(x) = \int_0^\infty F(t)\, G_n(x-t)\, dt \qquad x = T \tag{5}$$

$$K_n = \int_0^{2T} F(t)\, h_n(t)\, dt \tag{3b}$$

It becomes apparent, that over the range $0 \leq t \leq 2T$, $C_n' = K_n$ provided $h_n(t) = G_n(T-t)$. When $h_n(t)$ is a sine function

$$\sin\left(\frac{n\pi}{T}\right) t = \sin\left[\pi + \left(\frac{n\pi}{T}\right)(-t)\right] \qquad \begin{matrix} n = \text{odd and} \\ T = \pi \end{matrix} \tag{6}$$

$\therefore \quad K_n = C_n'(T)$ for odd sine functions.

When $h_n(t)$ is a cosine function

$$\cos\left(\frac{n\pi}{T}\right) t = -\cos\left[\pi + \left(\frac{n\pi}{T}\right)(-t)\right] \tag{7}$$

$\therefore \quad K_n = -C_n'(T)$ for odd cosine functions.

Taking into account the above results, one can substitute $G_n(T-t)$ for $h_n(t)$ in equation (3c) and obtain the Fourier series of $F(t)$ by changing the sign of the coefficients of the odd cosine functions in the series. It should be noted we have shown that the convolution integral can be used to obtain the Fourier coefficients of an odd periodic wave at time T, where T is the half

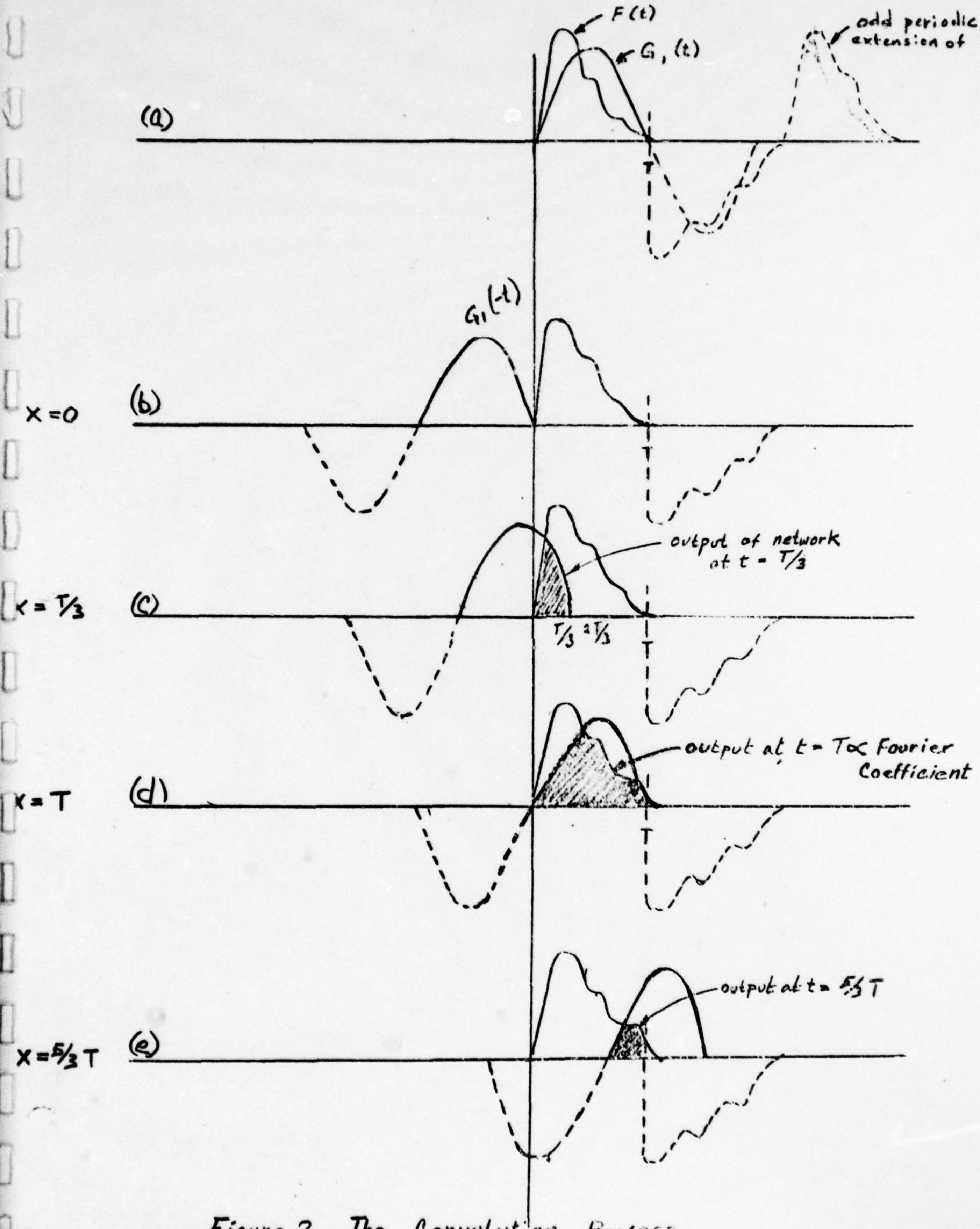


Figure 2 The Convolution Process

period of the fundamental sine and cosine functions of the Fourier series.

The problem, now, is to construct a network with an impulse response function $G_n(t)$, which will yield a Fourier coefficient of a given finite input function, $F'(t)$, as an odd periodic extension $F(t)$, as shown in Figure 2(a), and analyse only the first T seconds of the wave. It is easy to show that

$$\frac{1}{T}\int_0^T F'(t)\, G_n(x-t)\, dt = \frac{1}{2T}\int_0^{2T} F(t)\, G_n(x-t)\, dt \qquad (8)$$

when $F(t)$ is the odd periodic extension of $F'(t)$, i.e., it is necessary to analyse the finite function $F'(t)$ for only half the period of its odd periodic extension to determine the Fourier coefficients of the odd periodic extension of $F'(t)$.

It should be noted that $F'(t)$ will be analyzed as an odd periodic wave of period 2T by the proposed network. regardless of the duration of $F'(t)$, therefore it is important to insure that all significant information carried in $F'(t)$ occur within a time interval such that $F'(t)$ exists for $0 \leq t \leq T$. However, $F'(t)$ may exist for an interval greater than T, provided the information desired lies within the period $0 \leq t \leq T$, such a case is illustrated in Figures 2(a).

Consider the two series connected circuits shown in Figure 3(a)[2]. Let us assume the input current wave $G_n(t)$ to be a sine wave of period 2T. This input current produces a voltage e_1 from network 1, and a voltage e_2 from network 2 as shown in Figure 3(b) and 3(c). e_1 is a unit impulse voltage and e_2 a differentiated square wave voltage of unit amplitude. If the networks N_1 and N_2 are linear passive networks a voltage $e_o = e_1 + e_2$ can be applied

(2) The method described here is from Guillemin-Synthesis of Passive Networks, 1957, pgs. 717-726.

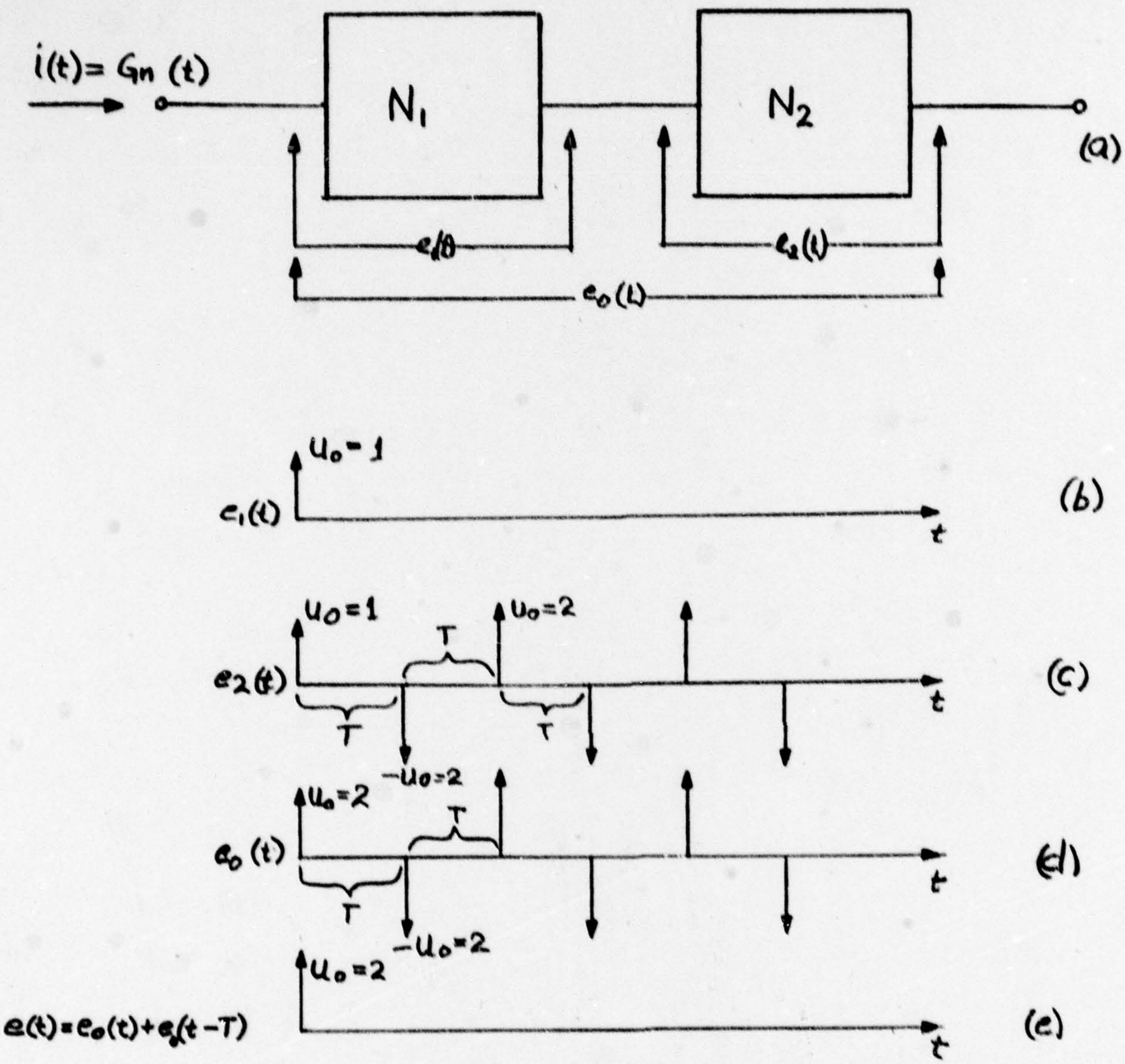


Figure 3 Network for producing an impulse response Gn (t)

across the series combination of N_1 and N_2 to produce the current wave $G_n(t)$. Now suppose a voltage $e(t) = e_o(t) + e_o(t-T)$ is applied to the network combination, Figures 3(d) and 3(e). One notes that $e(t)$ is nothing more than a single impulse of two units magnitude and that the output current, $G_n(t)$ is merely the positive half of a sine wave, but this is the wave form needed to determine the fundamental Fourier coefficient A_1 for the finite function $F'(t)$. Therefore, if a network can be synthesized with the transfer functions

$$T_n = \frac{(2)\ G_n(t)}{e_1(t) + e_2(t)} \qquad n = 1, 2, 3, \ldots \tag{9}$$

the Fourier coefficients of $F'(t)$ can be obtained from the networks. The numerator in equation (9) is multiplied by 2 so that a unit impulse $1U_o(t)$ will produce T_n instead of $2U_o(t)$.

One is now faced with synthesizing equation (9) into a realizable passive network. This can be accomplished by taking the Laplace transform of T_n to obtain a ratio of finite polynomials in the variable s. Since the problem under consideration requires $G_n(t)$ to be the sine and cosine terms of a Fourier series, it is best to express $G_n(t)$ as the Laplace transform of the general expression for the Fourier series of an odd function,

$$\mathcal{L} \sum_{\substack{n = \text{odd} \\ \text{integers}}}^{\infty} \left(A_n \sin \frac{n\pi}{T} t + B_n \cos \frac{n\pi}{T} t\right) = \sum_{\substack{n = \text{odd} \\ \text{integers}}}^{\infty} \frac{A_n + S\ B_n}{S^2 + \left(\frac{n\pi}{T}\right)^2} \tag{10}$$

The Laplace transform of $e_2(t)$, for the purpose of synthesis, can be written as the Laplace transform of the Fourier series of a differentiated unit square wave,

$$\mathcal{L}\ e_2(t) = S \sum_{\substack{n = \text{odd} \\ \text{integers}}}^{\infty} \frac{4/T}{S^2 + \left(\frac{n\pi}{T}\right)^2} \tag{11}$$

The Laplace transform of $e_1(t)$ is simply

$$\mathcal{L} e_1(t) = 1. \tag{12}$$

Equation (9) can now be expressed as

$$T'_n(S) = \frac{Z \sum\limits_{n=\text{odd}}^{j} \frac{A_n + S\,B_n}{S^2 + \left(\frac{n\pi}{T}\right)^2}}{1 + S \sum\limits_{n=\text{odd}}^{j} \frac{4/T}{S^2 + \left(\frac{n\pi}{T}\right)^2}} \tag{13}$$

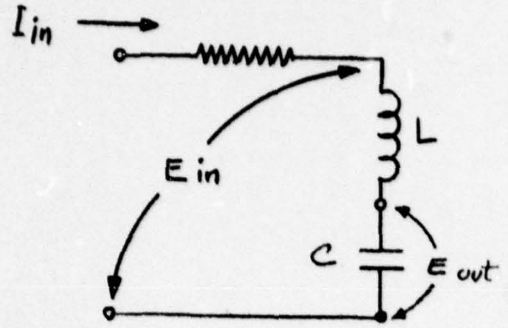
where the infinite series for the square wave has been truncated to a finite number of terms, j, to allow synthesis of $T_n(s)$.

One method of realizing equation (13) is to employ a negative feedback network. The sine functions in the nemerator of equation (13) can be generated (see Figure 4) as the impulse response of networks with voltage transfer functions of the form

$$T(S) = \frac{(1/LC)}{S^2 + (1/LC)} \,. \tag{14}$$

This transfer function has the same form as the Laplace transform of a sine wave with a frequency of 1/LC .

If a group of networks having transfer function of the form in equation (14), and time base outputs which are odd multiples of the time base 2T, are connected in parallel, the sine function outputs can be combined in an adder such that the output voltage will be a square wave (the square wave in equation (13) is a summation of odd sine wave functions). The output square wave voltage can then be differentiated and fed back to the input as negative feedback, thus producing finite sine functions at the output (see Figure 5). Writing the resultant voltage transfer expression for the system in Figure 5,



$$E_{out}(S) = \frac{E_{in}(S)}{SL + \frac{1}{CS}} \cdot \frac{1}{CS}$$

$$\frac{E_{out}(S)}{E_{in}(S)} = T(S) = \frac{(1/LC)}{s^2 + (1/LC)}$$

Figure 4 Network for generating sine and cosine functions

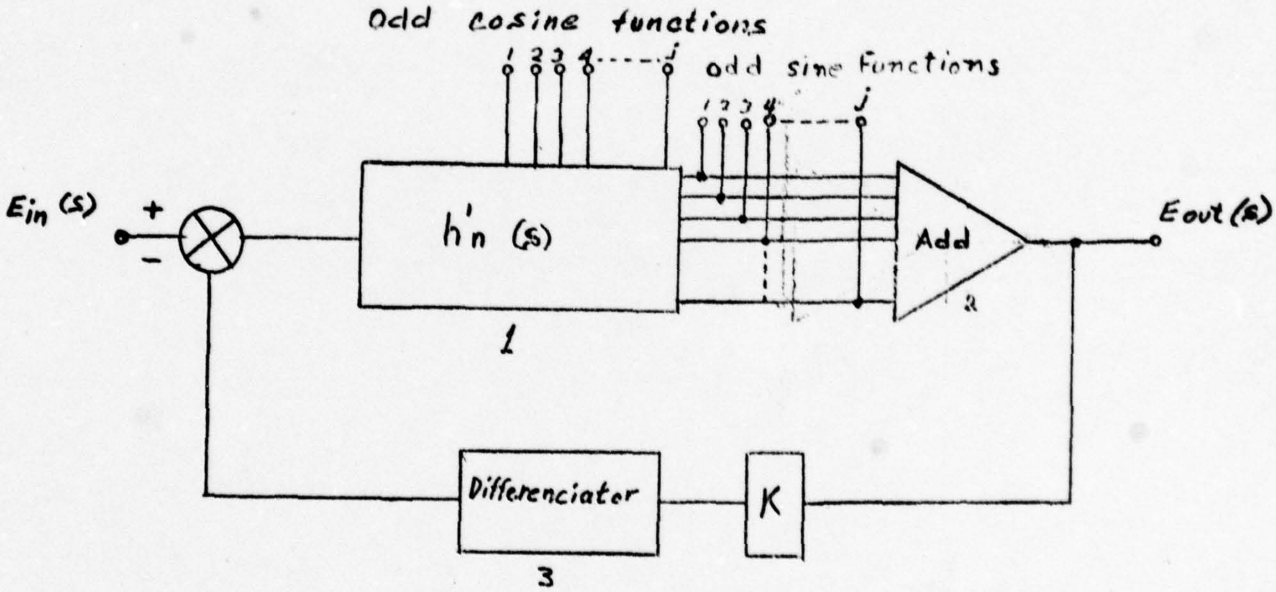


Figure 5 Feedback network for realizing equation (13).

$$E_{out}(S) = E_{in}(S) \sum_{n=odd}^{j} h'_n(S) - S[K E_{out}(S) \sum_{n=odd}^{j} h'_n(S)]$$

$$T'_n(S) = \frac{E_{out}(S)}{E_{in}(S)} = \frac{\sum_{n=odd}^{j} h'_n(S)}{1 + S[K E_{out}(S) \sum_{n=odd}^{j} h'_n(S)]}$$

Where:

$h'_n(s)$ = Laplace transform of network whose impulse responses are odd sine wave voltages.

K = constant.

This is the form of equation (13), except that we may also require cosine functions in the numerator. Noting that the current through capacitor has the same waveform as the voltage across the capacitor except shifted 90^{o} in phase, it is a simpler matter to convert the current in the sine function circuits to cosine voltages. One can also obtain the cosine functions by differentiating the sine function outputs. If we tap off the sine and cosine functions from box 1 in Figure 5, they can be fed to variable gain amplifiers and summed to form a finite function $F'(t)$ when the input, E_{in} is an impulse voltage; or the output functions from box 1 can be used to obtain the Fourier coefficients of the finite function $F'(t)$ when E_{in} is the function $F'(t)$.

III. Description of the Finite Fourier Filter Unit Proposed by Melpar

Figure 6 shows the system diagram of the finite Fourier filter unit proposed by Melpar. The principle of operation is the same as the system in Figure 5, section II; however, the unit proposed by Melpar will employ active R-C networks instead of passive R-L-C networks, and the odd cosine functions will be obtained by differentiating the odd output sine functions.

Description of Operation

Odd sine wave voltages are obtained from filters F_1 to F_6 when an impulse is applied to their inputs. These output sine functions are weighted, summed, and differentiated in block A_3 to form a differentiated square wave. The output of block A_3 is fed back to the parallel inputs of F_1 to F_6 as negative feedback. A square wave generator, block A_5, is used in conjunction with the differentiator of block A_3 to provide positive and negative impulses. These impulses activate the filters when they are used as a function generator.

With the feedback loop connected, the outputs of the filters become finite functions with a time duration of T. These finite outputs are tapped off F_1 to F_6 and fed to gain switches 1 to 6. Each gain switch contains a pair of two section wafer switches connected in parallel. The outputs from one of each pair of switches are summed in the bipolar adder, block A_4, to form a sum of odd sine wave functions. The other outputs from the gain switches are differentiated and summed in the bipolar adder to form a sum of odd cosine functions. The odd sine and cosine functions are combined in block A_4 to form a single output. The unit proposed by Melpar will also include separate outputs for each sine and cosine function; thus when an input function is applied to the filters, instead of impulses, the outputs will provide the

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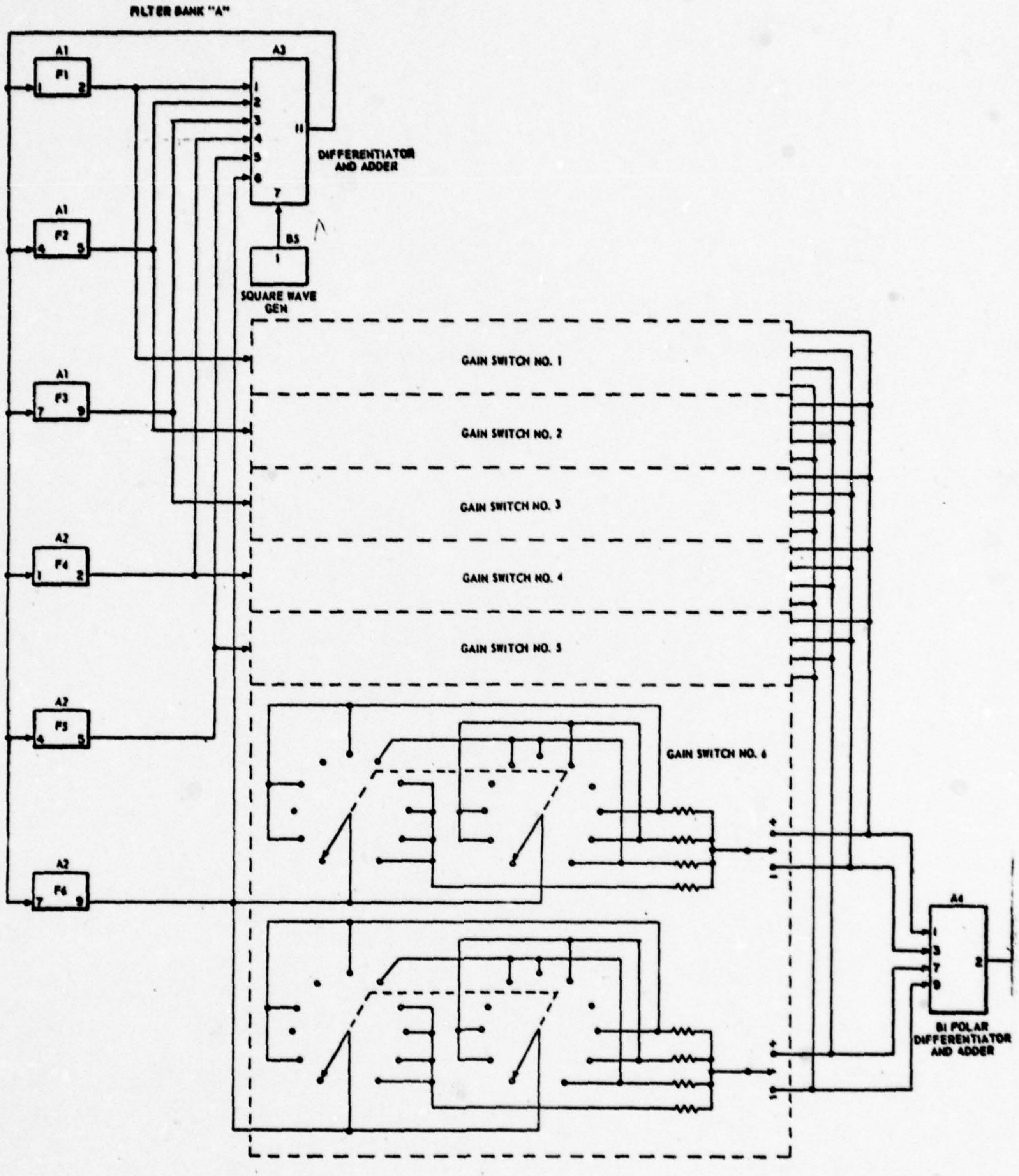


Figure 6 System Diagram of Finite Fourier Filter Unit
Proposed by Melpar

individual and combined sum of the Fourier coefficients of the past T seconds of the input function at time T.

IV. APPLICATIONS

From the previous sections it is apparent that finite Fourier filters can be applied to a variety of analysis and identification problems. Two particular applications, identification by modeling, and adaptive matched filters, have been studied by Melpar and will be presented in the following paragraphs.

(A) Adaptive Matched Filter

It can be shown(3) that, in the presence of white noise, the maximum signal to noise ratio at the output of a network occurs when the network is matched to the input signal. A network is matched to a given signal $f(t)$ if its impulse response is $f(x-t)$. From previous considerations it was found that a set of finite Fourier filters can generate a desired finite function $F'(t)$ when an impulse is applied to their inputs; thus, it is possible to match a set of Fourier filters to a given function $f(t)$ if $F'(t) = f(x-t)$.

Application of matched-filter techniques to communications and radar are well known. If the signal structure is not well known, in the sense that the transmitted signal has been distorted in some unpredictable manner, then the filters matched to idealized signals will no longer be perfectly matched to be actual received signals. Thus, one anticipates a deterioation from optimum considerations. Furthermore, there is no guarantee that the noise source will be white.

The technique described herein provides an "adaptive" filter which tracks a signal's structure so as to maintain maximum signal-to-noise ratio. This

(3) An Introduction to Matched Filters, G. L. Turin, IRE Transactions on Information Theory, June 1960, pp. 312.

is accomplished by what is described as Melpar's Adaptive Matched Filter Tracking System.

The single-input, multiple-output linear network, described in Section III, has each of its outputs connected to a summing amplifier through variable gain input networks. The response of this network can be made to approximate an arbitrary waveform $F'(t)$ within a time interval $(0,T)$. Changes in the network's response characteristics are accomplished by changing the input gains on the summing amplifier.

The value of the specific adaptive matched filter described above stems from a theorem showing that each of the inputs to the summing amplifier can be independently adjusted for maximum signal-to-noise ratio, so as to arrive at a match for a given input signal. That is, the process of matching the filter to a given signal can be accomplished by adjusting each input gain once, to achieve maximum signal-to-noise ratio.

Application of the adaptive matched-filter system for tracking low-level signals is illustrated in Figure 7. Signal inputs are continuously processed by the "real-time analyzer" network, providing M output components. These outputs are fed to three parallel circuits, each of which consists of a set of input networks (controllable) and a summing amplifier. Channel 1 is the matched-filter system. Channels 2 and 3 are representative experimental matched-filter systems, where one of the input gains has been perturbed so as to obtain a small increase in one and a small decrease in the other. The outputs of channels 2 and 3 are compared (from a timing signal generated from the output of channel 1) to establish the appropriate change in that input gain. This is transferred to all three channels, and the process sequenced to the next input.

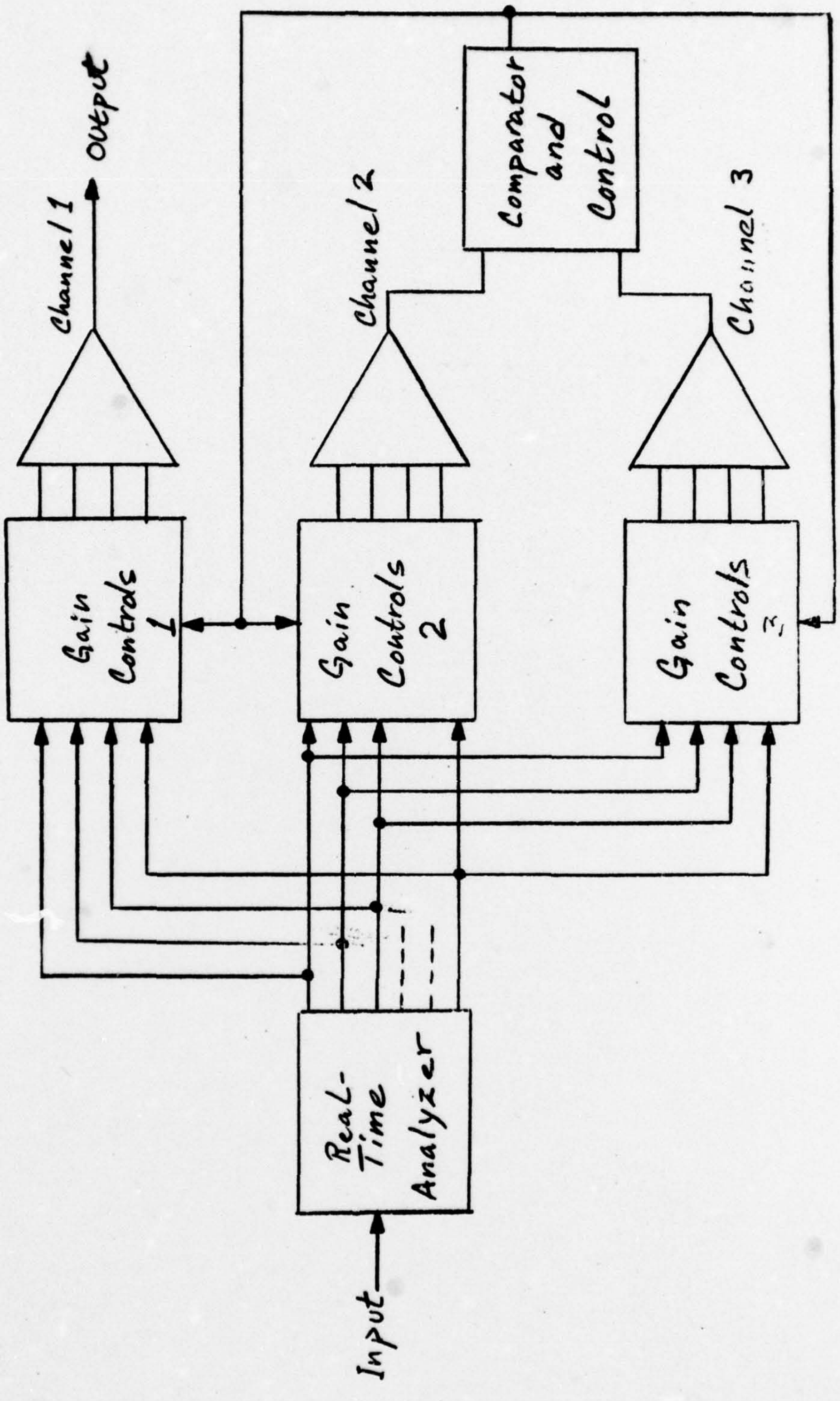


Figure 7. Matched-Filter Tracking Network

The time required for the system to sequence through each of the M input gains is the same as the time required to receive M inputs. This can be viewed as the tracking system's time constant. By increasing the number of experimental channels, it is possible to proportionately decrease this time constant. This type of system should provide improved communications whenever signal properties change in a relatively continuoys fashion, and where the signal distortions are not accurately predictable.

(B) Modeling

Modeling is the technique which attempts to duplicate a specific system for the purpose of analysis. Important applications of modeling exist where it is impossible or undesirable to disturb a system which is in operation. These applications are found in such areas as system identification and experimentation for modification of input variables.

Figure 8 shows a block diagram for modeling a linear system. The problem is to construct a model whose impulse response, $f'(t)$, is the same as the unknown linear system. Once this is determined, it is a simple matter to write down the transfer function of the unknown system. The model consists of a set of m Fourier filters whose outputs, Zm, are fed to a summing amplifier with variable gain inputs. When the output of the summing amplifier exactly matches the output of linear system the impulse response of the unknown linear system and the model will be the same. Practically, one can only hope to obtain a good approximation of the unknown system; however, the greater number of filters, m, the better the approximation. It can be readily shown[4] that the best approximation for a given set of filters will be obtained when the root mean

(4) Melpar Second Triannual Report, W3112.01, Machine Intelligence and Adaptive Systems, pp. 52-59.

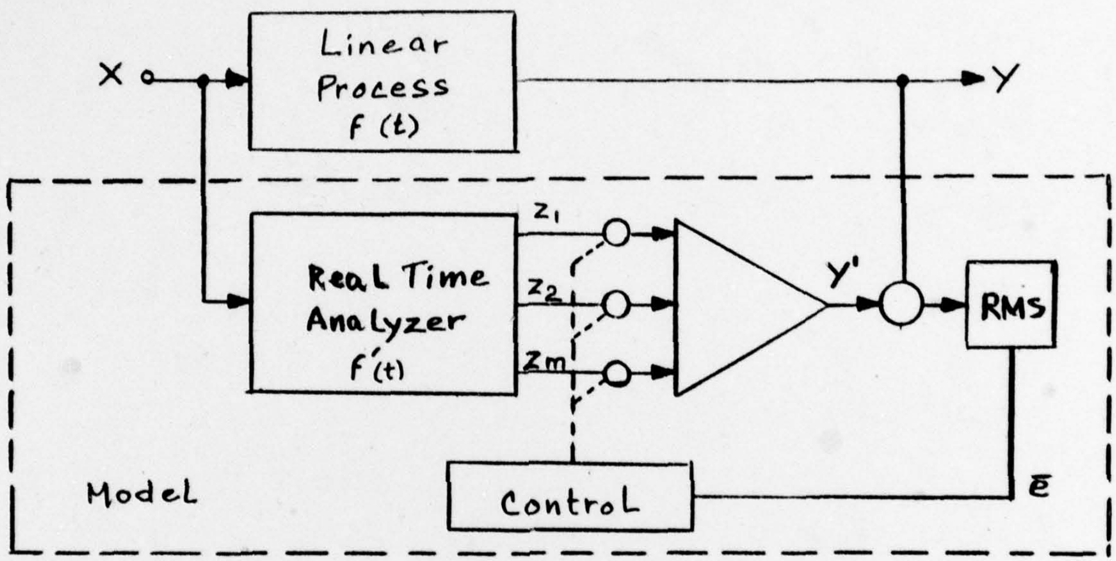


Figure 8. Identification with the Real-Time Analyzer

square difference in outputs, e, is minimized, i.e.,

$$\frac{d\overline{e^2}}{df'(t)} = 0 \text{ where } e = y-y'.$$

Further, since the impulse response is determined only by the gain settings of the input amplifiers of the summing amplifier, $\bar{e}$ can be used as a feedback voltage to converge the impulse response of the model to f(t). This implies that the variable gain elements in the model can be independently adjusted with respect to one another. If the variable gain elements remain independent over a specific range, the modeling system will be able to track a nonstationary linear system in that range; providing, of course, that the tracking time constant of the model is negligible compared to the rate of change in the linear system.

APPENDIX

Demonstration of an Adaptive Matched Filter Breadboard

Equipment was built to demonstrate the following:

a. The ability to approximate any desired finite duration waveform from impulse response from linear networks.

b. The application of this concept to form an adaptive matched filter.

c. The use of the active R-C synthesis technique, recently developed at Melpar.

The breadboard consists of two basic networks which are essentially identical in nature. One of these functions is to accomplish the approximation described in (a) above. The second of these networks corresponds to the adaptive matched filter as in (b) above.

The approximation and adaptive matched-filter concepts have been previously reviewed. It corresponds, essentially, to the odd harmonic Fourier approximation. The approximating terms (impulse responses of sub-networks) were each of finite duration T. In the breadboard, this time-duration was selected as 6.25 milliseconds. This long duration was selected to illustrate the application of the new, active R-C synthesis technique to circuits whose L-R-C realization would require large inductors.

A simplified block diagram of the breadboard is provided in Figure A-1. It illustrates the design concept rather than the design itself. Positive and negative impulses (obtained from the differentiation of a square wave are fed to filter bank A. The resulting outputs of filter bank A are the first, third, fifth, seventh, ninth, and eleventh sine and cosine waveforms shown in Figures A-2A through A-2L. It is noted that the higher order

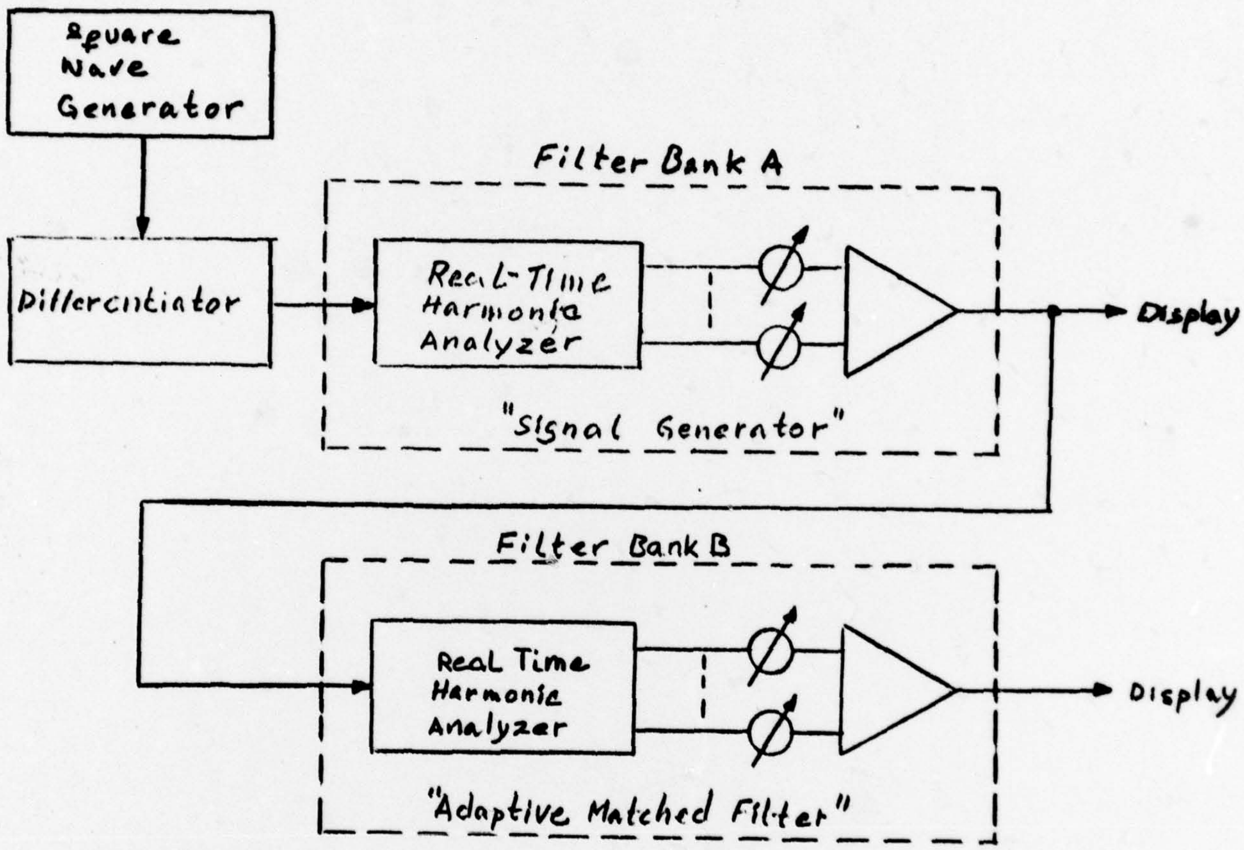


Figure A-1. Simplified Block Diagram of Breadboard

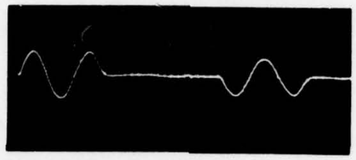
harmonics begin to deviate from the ideal, due to the absence of higher order terms. The cosine terms appear to be somewhat more noisy, but are superior to what could be reasonably expected. Linear combinations of these response functions were used to generate various waveforms.

The second function of the equipment is to provide a filter which is matched to the signals being generated. Although the matched filter was adjusted by hand, the application of appropriate circuitry for an adaptive matched filter is obvious. An example of an adaptive matched filter system was given in Figure 5, section IV. In Figure A-3, the ninth sine harmonic was generated (upper trace) and fed into a filter matched to it. The output (lower trace) is seen to be a close approximation of the anticipated autocorrelation of the compactly carried ninth harmonic. In Figure A-4, a square wave was generated (upper trace) and fed into a filter matched to it (lower trace) with the anticipated results.

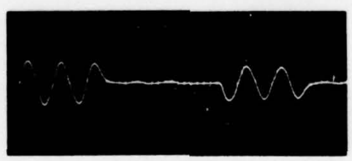
The behavior of the matched filter in the presence of noise is quite dramatic. This is illustrated in Figures A-5A through A-5D. The upper traces correspond to the signal input to the matched filter. For Figures A-5C and A-5D (upper trace), the scope sensitivity is 2 volts/cm. In each of these, the sensitivity of the lower trace was set at 0.5 volt/cm. Noise was obtained from a "sounvister" white noise diode and added to the generated signals. Its characteristics are essentially Gaussian from 2 cps to 100 mc. As shown, the matched-filter output can readily detect the presence of the signal.

The above experiment was repeated with an asymmetric waveform fed to a filter matched to it (triangular waveform). The results are illustrated in Figures A-6A through A-6C. This was as anticipated.

A First Harmonic (Sine)



B Third Harmonic (Sine)



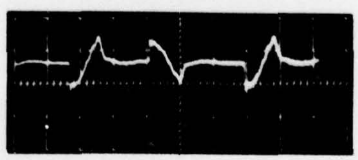
C Fifth Harmonic (Sine)

D Seventh Harmonic (Sine)

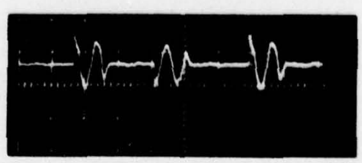
Figure A-2

E Ninth Harmonic (Sine)

F Eleventh Harmonic (Sine)



G First Harmonic (Cosine)

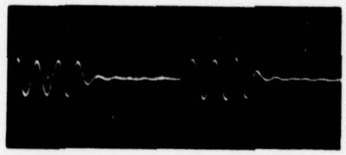


H Third Harmonic (Cosine)

Figure A-2

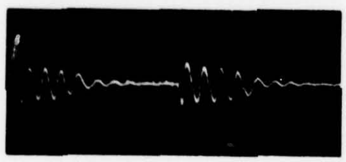
I

FIFTH HARMONIC (COSINE)



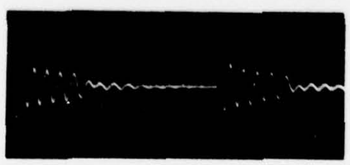
J

SEVENTH HARMONIC (COSINE)



K

NINTH HARMONIC (COSINE)



L

ELEVENTH HARMONIC (COSINE)

Figure A-2

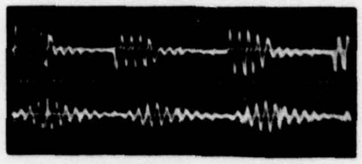


Figure A-3. Ninth Harmonic (Sine) - Top
Output of Filter Matched to the Ninth Harmonic - Bottom

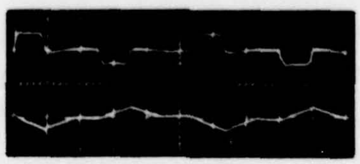
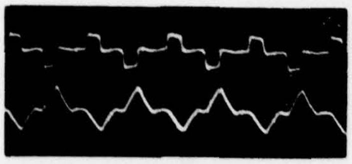


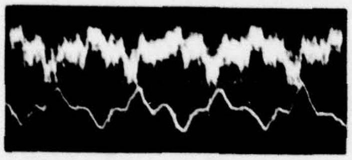
Figure A-4. Fourier Approximation of a Square Wave - Top
Output of Filter Matched to Square Wave - Bottom



A

SQUARE WAVE, 1 VOLT/CM

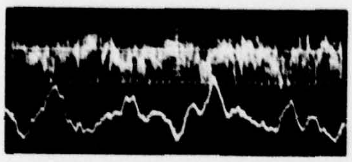
MATCHED FILTER OUTPUT, 0.5 VOLT/CM



B

SQUARE WAVE WITH WHITE NOISE, 1 VOLT/CM

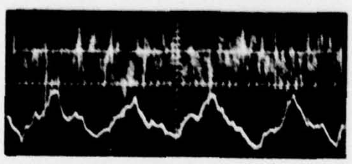
MATCHED FILTER OUTPUT, 0.5 VOLT/CM



C

SQUARE WAVE WITH INCREASED NOISE, 2 VOLTS/CM

MATCHED FILTER OUTPUT, 0.5 VOLT/CM



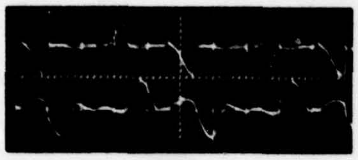
D

SQUARE WAVE WITH INCREASED NOISE, 2 VOLTS/CM

MATCHED FILTER OUTPUT, 0.5 VOLT/CM

Figure A-5

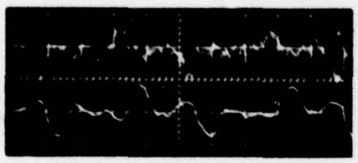
A



ASYMMETRIC WAVEFORM, 1 VOLT/CM

MATCHED FILTER OUTPUT, 0.5 VOLT/CM

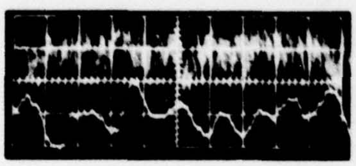
B



ASYMMETRIC WAVEFORM WITH NOISE, 1 VOLT/CM

MATCHED FILTER OUTPUT, 0.5 VOLT/CM

C



ASYMMETRIC WAVEFORM WITH INCREASED NOISE, 1 VOLT/CM

MATCHED FILTER OUTPUT, 0.5 VOLT/CM

Figure A-6